

PROTON DECAY IN SUPERSYMMETRIC FINITE GRAND UNIFICATION^{*}

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(August, 1994)

Abstract

We study proton decay in finite supersymmetric $SU(5)$ grand unified theories. We find that the finite supersymmetric $SU(5)$ models are ruled out from this consideration.

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^{*}Talk presented by X.-G. He at the Eighth Meeting the American Physical Society, Division of Particles and Fields (DPF'94), Albuquerque, New Mexico, August 2-6, 1994

Proton decays are predicted in many grand unified theories (GUTs) [1]. Experimentally no proton decays have been observed [2]. The stringent experimental bounds on proton decays can provide interesting constraints on GUTs [3–5]. It has been shown that in the minimal supersymmetric (SUSY) $SU(5)$ model, a large region in parameter space can be ruled out from this consideration [4,5]. Here we show that a class of finite SUSY $SU(5)$ model is ruled out by experimental bounds on the proton life-time [6].

There have been many studies of finite GUTs [7,8]. This is a class of interesting GUTs. It supports strongly the hope that the ultimate theory does not need infinite renormalization. In order to have a finite theory to all orders, the β functions for the gauge coupling and Yukawa couplings have to be zero to all orders. The requirement that the β function of the gauge coupling be zero greatly restricts the allowed matter representations in a theory. $\beta = 0$ for the Yukawa couplings can put additional constraints on the theory. A particularly interesting class of theories are the ones based on the $SU(5)$ gauge group with supersymmetry. If one requires that $SU(5)$ is broken by the Higgs mechanism to $SU(3)_C \times SU(2)_L \times U(1)_Y$ with three generations of matter fields, only one solution is allowed with 5, $\bar{5}$, 10, $\bar{10}$ and 24 chiral multiplets with multiplicities (4,7,3,0,1) [7,8]. This model contains one 24 (Σ) of Higgs for the $SU(5)$ breaking, $4(5 + \bar{5})$ (H_α , \bar{H}_α) of Higgs some of which will be used for electroweak breaking and the remaining $3(\bar{5} + 10)$ are identified with the three generation matter fields. With this content, the most general superpotential that may be written, consistent with renormalizability, $SU(5)$ invariance and R-parity conservation is of the form

$$W = qTr\Sigma^3 + MTr\Sigma^2 + \lambda_{\alpha\beta}\bar{H}_\alpha\Sigma H_\beta + m_{\alpha\beta}\bar{H}_\alpha H_\beta + \frac{1}{2}g_{ij\alpha}10_i10_jH_\alpha + \bar{g}_{ij\alpha}10_i\bar{5}_j\bar{H}_\alpha, \quad (1)$$

The indices α , β , and i , j run from 1 to 4 and 1 to 3, respectively.

The requirement that the β functions for the Yukawa couplings are zero at the one-loop level implies,

$$\begin{aligned} \Sigma : \quad & \frac{189}{5}q^2 = 10g^2 - \lambda_{\alpha\beta}\lambda^{\alpha\beta}, \\ \bar{H}_\alpha : \quad & \bar{g}_{ij\alpha}\bar{g}^{ij\beta} = \frac{6}{5}(g^2\delta_\alpha^\beta - \lambda_{\alpha\gamma}\lambda^{\beta\gamma}), \end{aligned}$$

$$\begin{aligned}
\bar{5}_i : \quad & \bar{g}_{k i \alpha} \bar{g}^{k j \alpha} = \frac{6}{5} g^2 \delta_i^j , \\
H_\alpha : \quad & g_{i j \alpha} g^{i j \beta} = \frac{8}{5} (g^2 \delta_\alpha^\beta - \lambda_{\gamma \alpha} \lambda^{\gamma \beta}) , \\
10_i : \quad & 2 g_{i k \alpha} g^{j k \alpha} + 3 g_{i k \alpha} g^{j k \alpha} = \frac{36}{5} g^2 \delta_i^j .
\end{aligned} \tag{2}$$

Imposing an additional $Z_7 \times Z_3$ symmetry [7], one obtains a unique solution to eq.(2):

$$g_{111}^2 = g_{222}^2 = g_{333}^2 = \frac{8}{5} g^2 , \quad \bar{g}_{111}^2 = \bar{g}_{222}^2 = \bar{g}_{333}^2 = \frac{6}{5} g^2 , \quad \lambda_{44} = g^2 , \quad q^2 = \frac{5}{21} g^2 . \tag{3}$$

All other tri-linear couplings are zero.

In the above model only $H_4(\bar{H}_4)$ can develop vacuum expectation values in order that the doublet-triplet mass splitting is possible for the doublets which break $SU(3)_C \times SU(2)_L \times U(1)_Y$ to $U(1)_{em}$. It is possible to find solutions such that each Higgs doublet can develop a vacuum expectation value and at the same time it is still possible to maintain the doublet-triplet mass splitting if the discrete symmetry is softly broken. All fermions can have masses [8]. Carrying out the renormalization group analysis from GUT scale to the electroweak scale, the top quark mass is found to be between 175 to 190 GeV [7]. This is a very interesting prediction.

There are, however, several problems with this model. Because the Yukawa couplings are diagonal, all KM angles are zero. This problem can be solved by abandoning the diagonal solution to eq.(2). It is possible to find a solution of eq.(2) such that KM matrix can be reproduced. A possible solution is

$$\bar{g}_{i j \alpha} = \sqrt{\frac{6}{5}} g (\delta_{i,1} \delta_{\alpha,1} + \delta_{i,2} \delta_{\alpha,2} + \delta_{i,3} \delta_{\alpha,3}) V_{ij} \tag{4}$$

with all other couplings the same as in eq.(3). Here V_{ij} is the KM matrix. This model has the same predictions for the quark masses.

This model also predicts the wrong mass relations for the first two generations: $m_e = m_d$, $m_\mu = m_s$ at the GUT scale because there are only 5 and $\bar{5}$ Higgs representations to generate masses for quarks and charged leptons. If higher dimension operators are somehow allowed, this problem can be solved. For example, adding a $(10 \times \bar{5})(\Sigma \bar{H}_\alpha)$ term can correct the

wrong mass relations. This, however, is not the major problem. In the following we will show that even if we relax the conditions to allow the above additions to the theory, the model has another problem. It predicts too rapid a proton decay.

There are several mechanisms by which proton decays may be induced in SUSY SU(5) theories. The exchanges of heavy gauge bosons, exchange of scalar color triplets, and dimension-five operator induced by exchange s-particles can all lead to proton decays. The most significant contributions to the proton decays come from the dimension-five operator induced by exchanging color triplet higgsinos H_C and \bar{H}_C of H_α and \bar{H}_α [3–5] and wino in the loop. In the minimal SUSY SU(5) model, this mechanism is the dominant one and considerably restricts the allowed region in parameter space [5]. In the finite SU(5) model, experimental bounds on proton decays all but make these models unacceptable [6]. The four-fermion baryon number violating effective Lagrangian at 1 GeV can be written down explicitly as [5]

$$\begin{aligned}
L = & \frac{\alpha_2}{2\pi M_{H_C}} g_{ii\alpha} \bar{g}_{kk\alpha} V_{jk}^* A_S A_L \\
& \times [(u_i d'_i)(d'_j \nu_k)(f(u_j, e_k) + f(u_i, d'_i)) + (d'_i u_i)(u_j e_k)(f(u_i, d_i) + f(d'_j, \nu_k)) \\
& + (d'_i \nu_k)(d'_i u_j)(f(u_i, e_k) + f(u_i, d'_j)) + (u_i d'_j)(u_i e_k)(f(d'_i, u_j) + f(d'_i, \nu_k))] ,
\end{aligned} \tag{5}$$

where $d'_i = V_{il} d_l$; $f(a, b) = m_{\bar{w}}[m_{\bar{a}}^2 \ln(m_{\bar{a}}^2/m_{\bar{w}}^2)/(m_{\bar{a}}^2 - m_{\bar{w}}^2) - (m_{\bar{a}} \rightarrow m_{\bar{b}})]/(m_{\bar{a}}^2 - m_{\bar{b}}^2)$ is from the loop integral, and $m_{\bar{a}, \bar{b}}$ are the s-fermion masses, $A_S \approx 0.59$, $A_L \approx 0.22$ [5] are the QCD correction factors for the running from M_{GUT} to SUSY breaking scale and from SUSY breaking scale to 1 GeV, respectively, and the Yukawa couplings are evaluated at 1 GeV.

Because all $g_{ii\alpha}$ and $\bar{g}_{jj\alpha}$ are equal in the model we are considering, the dominant contributions to the proton decays will be the ones involving only particles in the first generation. The dominant baryon number violating decay modes are: $p \rightarrow \pi^+ \bar{\nu}_e$, $p \rightarrow \pi^0(\eta)e^+$, $n \rightarrow \pi^0(\eta)\bar{\nu}_e$, $p \rightarrow \pi^- e^+$.

Finally to obtain the life times of the proton and neutron, we employ the chiral Lagrangian approach to parametrize the hadronic matrix elements. We have

$$\begin{aligned}
\Gamma(p \rightarrow \pi^+ \bar{\nu}_e) &= 2\Gamma(n \rightarrow \pi^0 \bar{\nu}_e) = \beta^2 \frac{m_N}{32\pi f_\pi^2} |C(duu\nu_e)(1 + D + F)|^2, \\
\Gamma(n \rightarrow \eta \bar{\nu}_e) &= \beta^2 \frac{(m_N^2 - m_\eta^2)^2}{64\pi f_\pi^2 m_N^3} 3|C(duu\nu_e)(1 - \frac{1}{3}(D - 3F))|^2, \\
\Gamma(n \rightarrow \pi^- e^+) &= 2\Gamma(p \rightarrow \pi^0 e^+) = \beta^2 \frac{m_N}{32\pi f_\pi^2} |C(duue)(1 + D + F)|^2, \\
\Gamma(p \rightarrow \eta e^+) &= \beta^2 \frac{(m_N^2 - m_\eta^2)^2}{64\pi f_\pi^2 m_N^3} 3|C(duue)(1 - \frac{1}{3}(D - 3F))|^2,
\end{aligned} \tag{6}$$

where $D = 0.81$ and $F = 0.44$, which arise from the strong interacting baryon-meson chiral Lagrangian, $f_\pi = 132$ MeV is the pion decay constant, and m_N and m_η are the nucleon and η meson masses, respectively. The parameter β is estimated to be in the range [9] 0.03 GeV^3 to 0.0056 GeV^3 . The parameters $C(duu\nu)$ and $C(duue)$ are the coefficients of the operators $(du)(u\nu)$ and $(du)(ue)$ which can be read off from eq.(5). We have

$$\begin{aligned}
C(duu\nu_e) &= \frac{4\alpha_{em}^2}{\sin^4 \theta_W} \frac{\bar{m}_b \bar{m}_t}{m_W^2 \sin 2\beta_H} \frac{A_S A_L}{M_{H_C^1}} V_{ud}^2 V_{ud}^* (f(u, e) + f(u, d)), \\
C(duue) &= \frac{4\alpha_{em}^2}{\sin^4 \theta_W} \frac{\bar{m}_b \bar{m}_t}{m_W^2 \sin 2\beta_H} \frac{A_S A_L}{M_{H_C^1}} V_{ud} V_{ud}^* (f(u, e) + f(u, d)).
\end{aligned} \tag{7}$$

In the above we have used $g_{111}\bar{g}_{111} = g_2^2 \bar{m}_b \bar{m}_t / m_W^2 \sin 2\beta_H$ as a good approximation. Here the quark masses are at 1 GeV. $\tan \beta_H$ is the ratio of the vacuum expectation value of H_1 to that of \bar{H}_1 . It is predicted to be about 50. The top quark mass at 1 GeV \bar{m}_t is about 470 GeV [7]. Using these values, we obtain the partial life-times for some of the baryon number violating decays as

$$\begin{aligned}
\tau(p \rightarrow \pi^0 e^+) &\approx \tau(n \rightarrow \pi^0 \bar{\nu}_e) \approx 6 \times 10^{17} \times P \text{ years}, \\
\tau(p \rightarrow \pi^+ \bar{\nu}_e) &\approx \tau(n \rightarrow \pi^- e^+) \approx 3 \times 10^{17} \times P \text{ years}, \\
\tau(p \rightarrow \eta e^+) &\approx \tau(n \rightarrow \eta \bar{\nu}_e) \approx 2 \times 10^{18} \times P \text{ years},
\end{aligned} \tag{8}$$

where

$$P = \left(\frac{0.003 \text{ GeV}^3}{\beta} \right)^2 \left(\frac{M_{H_C}}{10^{17} \text{ GeV}} \frac{TeV^{-1}}{f(u, d) + f(u, e)} \right)^2. \tag{9}$$

Using $\beta = 0.003 \text{ GeV}^3$, we find that even if we allow m_{H_C} to be the same order as the Planck mass, these partial life-times are in contradiction with experiments if the factor

$I = TeV^{-1}/(f(u, d) + f(u, e))$ is of order one. There are two possible ways this problem can be avoided. One requires the wino mass to be larger than 10^8 TeV. Another forces the s-fermion masses to be much larger than the wino mass. The s-fermion masses have to be larger than 2×10^3 TeV for $m_{\tilde{w}} > 100$ GeV with $m_{H_C} = 10^{17}$ GeV. All these solutions require that SUSY be broken at a scale much much larger than a few TeV. However such solutions spoil the nice feature of solving the hierarchy problem that is the rationale for using SUSY theories in the first place. From these consideration, the model discussed above is either ruled out, or quite unattractive needing a large SUSY breaking scale. We expect this problem to arise in most finite theories of grand unification that allow proton decay.

This work is supported in part by Department of Energy No. DE-FG06-85ER40224.

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